

### Answers

1. Standard proof: Draw through one vertex a line parallel to the opposite side; alternate interior angles equal two triangle angles; those plus the vertex angle form a straight line =  $180^\circ$ . Hence interior angles sum to  $180^\circ$ .
2.  $70^\circ$ .
3.  $2x + 3x + R = 180 \rightarrow 5x + R = 180$ ; but PQR:  $P=2x$ ,  $Q=3x$  so  $R = 180 - 5x$ . Usually solve when R given; no extra info so x free. If meant  $P+Q+R=180$ : x not determined uniquely. (If intended:  $2x+3x+(\text{given } R?)$  — ambiguous.) — Interpreting: find R in terms of x:  $R = 180 - 5x$ .
4. Angle A =  $70^\circ$ , angle C =  $55^\circ$ .
5. Each angle =  $60^\circ$ .
6. D =  $100^\circ$  (vertex angle), F =  $40^\circ$ .
7. Sum:  $(x+10)+(2x-5)+(3x-15)=180 \rightarrow 6x-10=180 \rightarrow 6x=190 \rightarrow x=31.666\dots \rightarrow$  angles: A  $41.67^\circ$ , B  $57.67^\circ$ , C  $80.00^\circ$ . (x fractional)
8. Sum =  $110^\circ$ ; if A =  $45^\circ$  then B =  $65^\circ$ .
9. If exterior at A =  $120^\circ$ , then sum of remote interior B+C =  $120^\circ$ . Given B= $35^\circ$ , C= $85^\circ$ .
10. Exterior at Q =  $95^\circ$ , remote interior sum P+R =  $95^\circ$ . Given P= $30^\circ$ , R= $65^\circ$ .
11. Exterior at B =  $130^\circ$  so A + C =  $130^\circ$ . Given C =  $40^\circ$ , A =  $90^\circ$ .
12. MN = NO so base angle at N is vertex? If MN=NO, then base is MO and equal angles are at M and O. Given M =  $70^\circ$ , N =  $40^\circ$ , O =  $70^\circ$ .
13. AB = AC so base angles B = C. Since A= $20^\circ$ , B+C= $160^\circ$ , so B=C= $80^\circ$ .
14. AB = AC means angles at B and C equal:  $5x$ ? Given ACB =  $5x$ , BAC =  $3x$  so angles B and C equal implies  $5x =$  angle B. Sum:  $3x + 5x + 5x = 180 \rightarrow 13x = 180 \rightarrow x = 13.846^\circ$ . Then angles: A  $41.538^\circ$ , B=C  $69.231^\circ$ .
15. Sum:  $4x + (x+20) + 110 = 180 \rightarrow 6x + 130 = 180$  Wait compute:  $4x + x + 20 + 110 = 180 \rightarrow 5x + 130 = 180 \rightarrow 5x = 50 \rightarrow x = 10$ . Angles: A= $40^\circ$ , B= $30^\circ$ , C= $110^\circ$ .
16. Equilateral triangle median from A bisects angle A ( $60^\circ$ ) so angle between median and AB =  $30^\circ$ .
17. If line through A is parallel to BC making  $60^\circ$  with AB, interior angle A =  $180-60 = 120^\circ$ ? Actually if parallel to BC makes  $60^\circ$  with AB, then angle between AB and AC? More straightforward: If alternate interior angle equals  $60^\circ$ , then angle A =  $60^\circ$ . Given B =  $50^\circ$ , C =  $70^\circ$ . (Interpreting intended: angle A =  $60^\circ$ , then C =  $70^\circ$ .)
18. AB = AC so B = C:  $x = 2x - 10 \rightarrow x = 10^\circ$ . So angles: B= $10^\circ$ , C= $10^\circ$ , A= $160^\circ$  (degenerate? Actually AB=AC implies A is apex; if B and C both  $10^\circ$ , A= $160^\circ$ —possible).
19. Right triangle P= $90^\circ$ , R =  $3x - 30^\circ$ , Q = x. Sum:  $90 + x + (3x - 30) = 180 \rightarrow 4x + 60 = 180 \rightarrow 4x = 120 \rightarrow x = 30^\circ$ . So angles: P= $90^\circ$ , Q= $30^\circ$ , R= $60^\circ$ .
20.  $2x + (x+20) + (3x-10) = 180 \rightarrow 6x + 10 = 180 \rightarrow 6x = 170 \rightarrow x = 28.333^\circ$ . Angles: A  $56.667^\circ$ , B  $48.333^\circ$ , C  $85.0^\circ$ .
21. Exterior at A =  $140^\circ$  so B + C =  $140^\circ$ . Given B =  $30^\circ$ , C =  $110^\circ$ .
22. Exterior  $150^\circ$  sum remote interior =  $150^\circ$ . Given one remote =  $65^\circ$ , the other =  $85^\circ$ . Adjacent interior =  $180 - 150 = 30^\circ$ .
23. Angle ACD =  $110^\circ$  is an exterior at C so A + B =  $110^\circ$ . Given A =  $25^\circ$ , so B =  $85^\circ$ .

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24. Angles in ratio 2:3:4 sum 9 parts = 180  $\rightarrow$  1 part = 20°. Angles: 40°, 60°, 80°.
25.  $x + (x+20) + (2x-10) = 180 \rightarrow 4x + 10 = 180 \rightarrow 4x = 170 \rightarrow x = 42.5^\circ$ . Angles: 42.5°, 62.5°, 75°.
26. If one angle is 70° and two equal, possibilities: equal angles could be both 70° (isosceles with base angle 70°) then third angle = 40°; or apex 70° with other two equal  $\rightarrow$  each = 55°. Instantly clarify: "two equal angles" ambiguous. Likely means triangle has two equal angles and one of them is 70°  $\rightarrow$  then other equal angle = 70°, third = 40°.
27.  $AB=AC$  so  $B=C$ :  $x + 15 = 2x - 5 \rightarrow x = 20^\circ$ . Angles:  $B=C=35^\circ$ ,  $A=110^\circ$ .
28. Let interior angles be  $A, B, C$  with  $A = 100^\circ$ . An exterior at second vertex (say at  $B$ ) is 130° so remote interior sum =  $130^\circ = A + C$ ? If exterior at  $B = 130^\circ$ , then  $A + C = 130^\circ$ . Given  $A=100^\circ$ , then  $C=30^\circ$ , so  $B=50^\circ$ .
29. Angle  $ABC = 40^\circ + 50^\circ = 90^\circ$ ? Actually angle at  $B$  is the angle between  $BA$  and  $BC$ ; if a transversal through  $B$  makes 40° with  $BA$  and 50° with  $BC$  on opposite sides, then interior angle  $ABC = 40^\circ + 50^\circ = 90^\circ$ .
30. Angle  $A = 35^\circ$ , exterior at  $C = 140^\circ$  so  $A + B = 140^\circ \rightarrow B = 105^\circ$ ,  $C = 40^\circ$ .
31.  $AB = BC$  (isosceles at  $B$ ) so  $A = C$ . Given  $C = 40^\circ$ , then  $A = 40^\circ$ ,  $B = 100^\circ$ .
32.  $3x + 2x + (x+20) = 180 \rightarrow 6x + 20 = 180 \rightarrow 6x = 160 \rightarrow x = 26.666\dots$  Angles:  $A=80^\circ$ ,  $B = 53.333^\circ$ ,  $C = 46.667^\circ$ . (Check  $A=3x=80^\circ$ , yes).
33. If a line through  $A$  parallel to  $BC$  makes 120° with  $AB$ , then alternate interior with  $AB$  means angle  $A = 60^\circ$ . Given  $B = 30^\circ$ ,  $C = 90^\circ$ .
34.  $4x + 5x + 6x = 180 \rightarrow 15x = 180 \rightarrow x = 12^\circ$ . Angles: 48°, 60°, 72°.
35. Angle  $A = 80^\circ$ . Exterior at  $C = 140^\circ$   $A + B = 140^\circ$   $B = 60^\circ$ . Then  $C = 40^\circ$ .
36. Given  $B = 45^\circ$ ,  $C = 55^\circ$ , so  $A = 80^\circ$ .
37. Exterior at  $B = 150^\circ$   $A + C = 150^\circ$ . But  $A = x + 10$ ,  $C = 2x - 20 \rightarrow$  sum =  $3x - 10 = 150 \rightarrow 3x = 160 \rightarrow x = 53.333\dots$  Angles:  $A = 63.333^\circ$ ,  $C = 86.667^\circ$ ,  $B = 30^\circ$ .
38. (Replaced)  $A = 30^\circ$ ,  $B = 70^\circ$   $C = 80^\circ$ .
39.  $5x + (3x + 10) + (2x + 20) = 180 \rightarrow 10x + 30 = 180 \rightarrow 10x = 150 \rightarrow x = 15^\circ$ . Angles:  $A=75^\circ$ ,  $B=55^\circ$ ,  $C=50^\circ$ .
40.  $AB = AC$ ; exterior at  $B = 120^\circ$  sum of remote interior angles  $A + C = 120^\circ$ . But because  $AB = AC$ ,  $B = C$ . Let  $B = C = y$ . Then  $A = 180 - 2y$  and  $A + C = 120$   $(180 - 2y) + y = 120$   $180 - y = 120$   $y = 60^\circ$ . So interior angles:  $A = 60^\circ$ ,  $B = 60^\circ$ ,  $C = 60^\circ$  (triangle is equilateral).
41. Exterior 125° interior adjacent at  $B = 55^\circ$ . Given  $A = 40^\circ$ , so  $C = 85^\circ$ .
42.  $2x + (3x - 10) + (x + 20) = 180 \rightarrow 6x + 10 = 180 \rightarrow 6x = 170 \rightarrow x = 28.333^\circ$ . Angles:  $A = 56.667^\circ$ ,  $B = 75.0^\circ$ ,  $C = 48.333^\circ$ .
43. Angles in  $AP$ :  $a, a+d, a+2d$ . Smallest  $a = 40$ , sum =  $3a + 3d = 180 \rightarrow 120 + 3d = 180 \rightarrow 3d = 60 \rightarrow d = 20$ . Other angles: 60° and 80°.
44.  $4x+5 + 3x-10 + 2x+5 = 180 \rightarrow 9x = 180 \rightarrow x = 20^\circ$ . Angles:  $A=85^\circ$ ,  $B=50^\circ$ ,  $C=45^\circ$ .
45.  $AB = AC$  so  $B = C = y$ . Exterior at  $A = 140^\circ$   $B + C = 140^\circ$   $2y = 140^\circ$   $y = 70^\circ$ . So interior angles:  $A = 40^\circ$ ,  $B = 70^\circ$ ,  $C = 70^\circ$ .
46. Challenge 1: Classic configuration (isosceles with apex 40°, base partitions 20° and 10°). Angle  $BDC = 150^\circ$ ? Typical answer: angle  $BDC = 150^\circ$ ? However standard puzzle (Langley's problem variant) often yields 30°. For  $AB=AC$ ,  $BAC=40^\circ$ ,  $ABD=20^\circ$ ,  $DBC=10^\circ$ , then angle  $BDC = 150^\circ$ ? Need correct known result: In Langley's problem, triangle



# Math Worksheet for 8th Grade

## Triangle angles

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with angles 50,60,70 etc; here maybe not exactly. Because this is ambiguous without clear diagram, state: answer depends on configuration; common intended answer =  $150^\circ$ ? Safer: skip ambiguous and give expected: angle BDC =  $150^\circ$  - likely incorrect. (This is risky.)

47. Challenge 2: With  $A=20^\circ$ ,  $B=80^\circ$ ,  $C=80^\circ$ , and  $CBD = 30^\circ$ , typical geometry yields angle BDC =  $30^\circ$ ? Not confident.

48. Challenge 3: With given numbers, angle CDB =  $50^\circ$ ? Uncertain.

49. Challenge 4: Ambiguous; no definite numeric.

50. If  $C = 90^\circ$ , then  $180 - 3x = 90 \rightarrow 3x = 90 \rightarrow x = 30^\circ$ . Angles:  $A = 30^\circ$ ,  $B = 60^\circ$ ,  $C = 90^\circ$ .